

# UK INTERMEDIATE MATHEMATICAL CHALLENGE

THURSDAY 3RD FEBRUARY 2005

Organised by the **United Kingdom Mathematics Trust**  
from the **School of Mathematics, University of Leeds**



## **RULES AND GUIDELINES** (to be read before starting)

1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.  
No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 11 or below.  
Candidates in Scotland must be in S4 or below.  
Candidates in Northern Ireland must be in School Year 12 or below.
5. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
6. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
7. Five marks are awarded for each correct answer to Questions 1-15.  
Six marks are awarded for each correct answer to Questions 16-25.  
**Each incorrect answer to Questions 16-20 loses 1 mark.**  
**Each incorrect answer to Questions 21-25 loses 2 marks.**
8. Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the sheet except to mark your chosen options**. The machine 'sees' all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.
9. The questions on this paper challenge you to **think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. The UK IMC is about solving interesting problems, not about lucky guessing.

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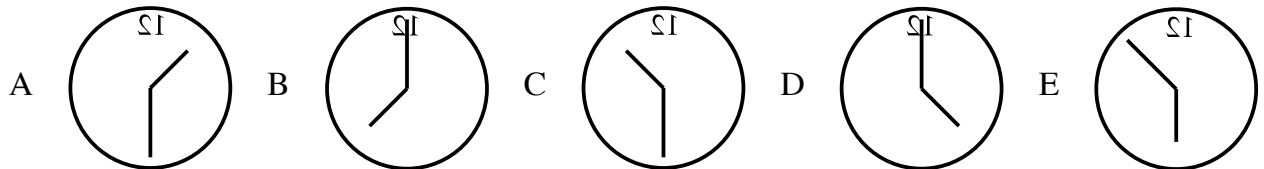
1. If the following numbers are arranged in increasing order of size, which one is in the middle?

- A 4.04                  B 4.004                  C 4.4                  D 4.44                  E 4.044

2. What is the difference between 10% of one million and 10% of one thousand?

- A 9 900                  B 9 990                  C 90 900                  D 99 900                  E 999 900

3. Professor Rosseforp has an unusual clock. The clock shows the correct time at noon, but the hands move anti-clockwise rather than clockwise. The clock is very accurate, however, so the hands move at the correct speeds. If you looked in a mirror at the Professor's clock at 1:30 pm, which of the following would you see?



4. Which of the following expressions is equal to 2005?

- A  $(1^2 + 1)(10^2 + 1)$                   B  $(2^2 + 1)(20^2 + 1)$                   C  $(3^2 + 1)(30^2 + 1)$   
 D  $(4^2 + 1)(40^2 + 1)$                   E  $(5^2 + 1)(50^2 + 1)$

5. How many of the statements in the box are true?

Any number which is divisible by 6 is even.	Any number which is divisible by 9 is odd.
The sum of any two odd numbers is even.	The sum of any two even numbers is odd.

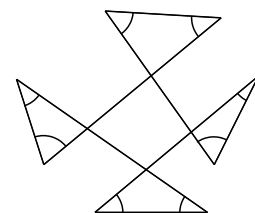
- A 0                  B 1                  C 2                  D 3                  E 4

6. A shop advertises 'Buy one, get one at half price'. For this offer, the average cost per item is the same as:

- A Two for the price of one                  B Three for the price of one                  C Three for the price of two  
 D Four for the price of three                  E Five for the price of four

7. In the diagram, what is the sum of the marked angles?

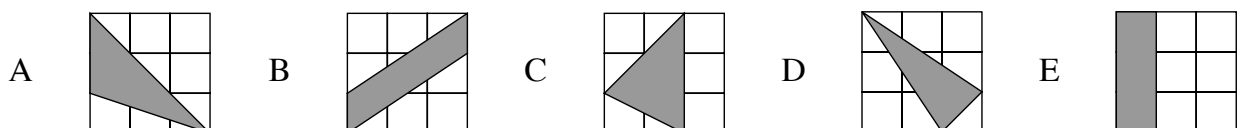
- A  $180^\circ$     B  $360^\circ$     C  $450^\circ$     D  $540^\circ$     E  $720^\circ$



8. What fraction of a 24-hour day does school take up, if school starts at 8:30am and finishes at 3:15pm?

- A  $\frac{9}{32}$                   B  $\frac{25}{96}$                   C  $\frac{13}{48}$                   D  $\frac{31}{96}$                   E  $\frac{18}{32}$

9. Which of the following shaded regions has an area different from the other shaded regions?



10. Granny has taken up deep-sea fishing! Last week, she caught a fish so big that she had to cut it into three pieces (head, body and tail) in order to weigh it. The tail weighed 9kg and the head weighed the same as the tail plus one third of the body. The body weighed as much as the head and tail together. How much did the whole fish weigh?

- A 18kg      B 27kg      C 54kg      D 77kg      E 84kg

11. If two of the sides of a right-angled triangle are 5 cm and 6 cm long, how many possibilities are there for the length of the third side?

- A 0      B 1      C 2      D 3      E 4

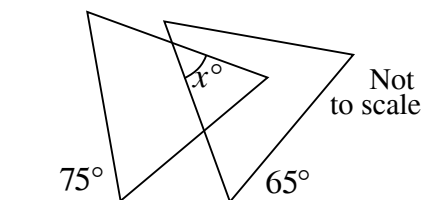
12. One gallon of honey provides enough fuel for a bee to fly about seven million miles. Roughly how many bees could fly one thousand miles if they had ten gallons of honey to share between them?

- A 7 000      B 70 000      C 700 000      D 7 000 000      E 70 000 000

13. The diagram shows two equilateral triangles.

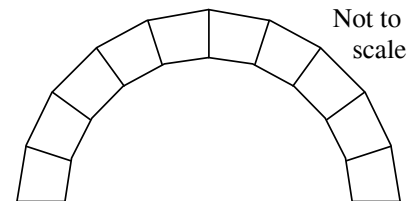
What is the value of  $x$ ?

- A 70      B 60      C 50      D 40      E 30



14. Ten stones, of identical shape and size, are used to make an arch, as shown in the diagram. Each stone has a cross-section in the shape of a trapezium with three equal sides. What is the size of the smallest angles of the trapezium?

- A  $72^\circ$       B  $75^\circ$       C  $81^\circ$       D  $83^\circ$       E  $85^\circ$

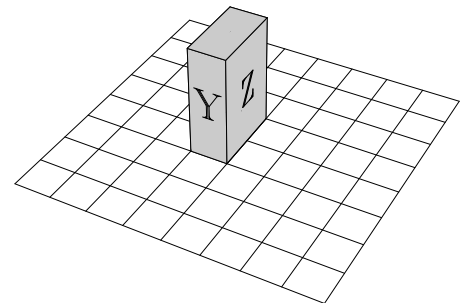


15. To make porridge, Goldilocks mixes together 3 bags of oats with 1 bag containing 20% wheat bran and 80% oats. All the bags have the same volume. What percentage of the volume of Goldilocks' porridge mixture is wheat bran?

- A 5%      B  $6\frac{2}{3}\%$       C 20%      D  $26\frac{2}{3}\%$       E 60%

16. A  $1 \times 2 \times 3$  block is placed on an  $8 \times 8$  board, as shown with the  $1 \times 2$  face 'X' at the bottom. It is rolled over an edge without slipping onto the  $1 \times 3$  face Y, then onto the  $2 \times 3$  face Z, then onto X, Y, Z again in that order. How many different squares on the board has the block occupied altogether, including the starting and ending positions?

- A 18      B 19      C 20      D 21      E 22

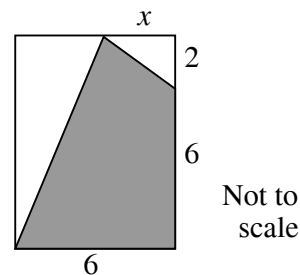


17. Platinum is a very rare metal, even rarer than gold. Its density is  $21.45 \text{ g/cm}^3$ . Assuming that the world production has been about 110 tonnes for each of the past 50 years, and negligible before that, which of the following has a comparable volume to that of the total amount of platinum ever produced?

- A a shoe box      B a cupboard      C a house  
D Buckingham Palace      E the Grand Canyon

18. Three-quarters of the area of the rectangle has been shaded.  
What is the value of  $x$ ?

A 2      B 2.4      C 3      D 3.6      E 4



19. Trinni is fascinated by triangle numbers (1, 3, 6, 10, 15, 21, etc.) and recently, coming across a clock, she found that she could rearrange the twelve numbers 1, 2, 3, ... 12 around the face so that each adjacent pair added up to a triangle number. She left the 12 in its usual place; what number did she put where the 6 would usually be?

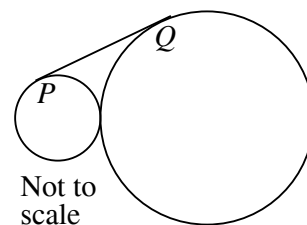
A 1                      B 4                      C 5                      D 10                      E 11

20. One of the following is the largest of nine consecutive positive integers whose sum is a perfect square. Which one is it?

A 118                      B 128                      C 138                      D 148                      E 158

21. Two circles with radii 1 cm and 4 cm touch. The point  $P$  is on the smaller circle,  $Q$  is on the larger circle and  $PQ$  is a tangent to both circles. What is the length of  $PQ$ ?

A  $\sqrt{17}$  cm              B 3 cm                      C  $2\sqrt{3}$  cm  
D  $3\sqrt{2}$  cm              E 4 cm



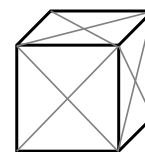
22. Inspector Remorse had a difficult year in 2004. A crime wave in Camford meant that he had 20% more cases to solve than in 2003, but his success rate dropped. In 2003, he solved 80% of his cases, but in 2004 he solved only 60% of them. What was the percentage change in the number of cases he solved in 2004 compared with 2003?

A Down by 10%      B Down by 8%      C No change      D Up by 8%      E Up by 10%

23. What is the area (in square units) of the triangle formed by the three lines whose equations are:  $y - x = 6$ ,  $x - 2y = 3$ ,  $x + y = 6$ ?

A 55                      B 60                      C 65                      D 70                      E 75

24. The figure shows a cube of side 1 on which all twelve face diagonals have been drawn – creating a network with 14 vertices (the original eight corners, plus the six face centres) and 36 edges (the original twelve edges of the cube plus four extra edges on each face). What is the length of the shortest path along the edges of the network which passes through all 14 vertices?



A  $1 + 6\sqrt{2}$       B  $4 + 2\sqrt{2}$       C 6                      D  $8 + 6\sqrt{2}$       E  $12 + 12\sqrt{2}$

25. This regular hexagon has been divided into four trapezia and one hexagon. If each of the five sections has the same perimeter, what is the ratio of the lengths  $p$ ,  $q$  and  $r$ ?

A 8 : 2 : 1              B 12 : 4 : 1              C 9 : 3 : 1  
D 6 : 3 : 1              E 9 : 4 : 1

